Unit 1: Probability

MBS 1st Semester Statistics Notes

Then the required permutation is

$$15_{P_3} = \frac{15!}{(15-3)!} = \frac{15!}{12!} = \frac{15\times 14\times 13\times 12!}{12!}$$

$= 15 \times 14 \times 13 = 2730$

Example 2

If five persons enter a bus in which there are 8 vacant seats, find in how many ways they can sit.

Solution:

Here, n = 8, r = 5

Number of arrangements of 4 persons in 8 seats

$$n_{P_r} = 8_{P_5} = \frac{8!}{(8-5)!} = \frac{8!}{3!} = 6720$$

∴ The four persons can sit in 8 seats by 6720 ways.

Example 3

How many plates of vehicles consisting of 4 different digits can be made out of the integers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9?

Solution:

Here, n = 10, r =4

Number of arrangements of 10 digits in four empty places

$$n_{P_r} = 10_{P_4} = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 5040$$

∴ Total number of plates of vehicles of 4 different digits that can be made = 5040

Combination (Definition)

A combination of n different objects taken r at a time is a selection of r objects out of n objects, without any regard to the order of arrangements.

The combinations of n different objects taken r at a time, denoted by ^nC_r or C (n, r) is given by

$$^nC_r = \frac{n!}{(n-r)!\,r!}; \quad r \leq n$$

Example 4

In an entrance test, 30 questions are set. In how many ways 25 questions can be chosen to answer.

Solution:

Here, n= 30, r = 25

Number of selections of 25 questions out of 30

$$^nC_r = {}^{30}C_{25} = \frac{30!}{(30-25)!\,25!} = \frac{30!}{5!\,25!} = 142506$$

∴ 25 questions can be chosen from 30 questions by 142506 ways.

Example 5

A bag contains 10 white balls and 8 red balls. In how many ways can 6 white and 5 red balls be drawn?

Solution:

Here, No. of white balls = 10

No. of red balls = 8

No. of selections of 6 white balls out of 10 balls and 5 red balls out of 8 balls = $^{10}C_6 \times {}^8C_5$

$= 210 \times 56$

$= 11760$

∴ Total number of ways of selections of 6 white and 5 red balls = 11760

1.4 Definitions of Probability

There are mainly four approaches to defining probability.

i) Mathematical or classical or a priori probability.
ii) Statistical or empirical or relative frequency probability.
iii) Subjective probability
iv) An axiomatic approach to probability.

1.4.1 Mathematical or Classical orPriori Definition of Probability

If a trial has n exhaustive, equally likely and mutually exclusive cases, out of which m are favourable to the happening of an event E, then the probability of happening of E is given by

$$P(E) = \frac{\text{Favourable number of cases to E}}{\text{Exhaustive number of cases}} = \frac{m}{n}$$

The probability of non-occurrence of the event E is given by

$$P(\overline{E}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(E)$$

$$\therefore \quad P(E) + P(\overline{E}) = 1 \quad \text{or,} \quad p + q = 1$$

Where p is called the probability of success and q is called the probability of failure. If P(E) = 1, E is called a certain event and if P(E) = 0, E is called an impossible event.

Limitations of Classical Definition

The classical definition of probability fails to solve the problem if

i) The number of exhaustive cases in a trial is infinite or unknown,
ii) The various outcomes of a trial are not equally likely.

For example, if a person jumps from the top of a multi starey building, then the probability of his/ her survival or death will not be 50%.

Example 6

A committee of 4 people is to be appointed from 3 economists, 4 engineers, 2 statisticians and 1 doctor. Find the probability of forming a committee in the following manner.

i) There must be one from each profession.

ii) It should have at least one economist and a doctor

Solution:

Here, total number of persons = 4 + 3 + 2 + 1 = 10

The exhaustive number of cases of 4 out of 10 = $^{10}C_4$

The favorable number of cases for each profession = $^{3}C_1 \times {}^{4}C_1 \times {}^{2}C_1 \times {}^{1}C_1$

So, the probability of selecting one each profession

$$= \frac{3_{C_1} \times 4_{C_1} \times 2_{C_1} \times 1_{C_1}}{10_{C_4}}$$

$$= \frac{\frac{3!}{1! \times (3-1)!} \times \frac{4!}{1! \times (4-1)!} \times \frac{2!}{1! \times (2-1)!} \times \frac{1!}{1! \times (1-1)!}}{\frac{10!}{10! \times (10-1)!}}$$

$$= \frac{24}{210}$$

$$= \frac{4}{35} = 0.1142$$

The probability that a committee consists of a doctor and at least one economist is given by

p = P (one doctor, one economist and two others) + P (one doctor, two economists and one other) + P (one doctor, three economists)

$$= \frac{1_{C_1} \times 3_{C_1} \times 6_{C_2}}{10_{C_4}} + \frac{3_{C_1} \times 3_{C_2} \times 5_{C_1}}{10_{C_4}} + \frac{1_{C_1} \times 3_{C_3}}{10_{C_4}}$$

= 0.3048

Example 7

A bag contains 3 red, 6 white and 7 black balls, what is the probability that two balls are drawn are white and black?

Solution:

Total number of balls = 3+6+7=16

Now, the probability of drawing 1white ball and 1 black ball is given by

$$P = \frac{{}^{6}C_1 \times {}^{7}C_1}{{}^{16}C_2} = 0.35$$

Example 8

In a group of an equal number of men and women, 30 % of men and 60 % of women are unemployed. What s the probability a person selected at random is employed?

Solution:

Assuming that the population of men and women being 100.

	Unemployed	Employed	Total
Men	30	70	100
Women	60	40	100
Total	90	110	200

If A be the event of selecting the employed person, then

$$P(A) = \frac{m}{n} = \frac{\text{Total employed person}}{\text{Total person}} = \frac{110}{200} = \frac{11}{20}$$

Example 9

The probability that a price of a commodity will rise is $\frac{1}{2}$, the probability will decrease is $\frac{1}{3}$. Find the probability that the price of the commodity will remain constant?

Solution:

Let, the probability that the price of the commodity will rise.

$$P(A) = \frac{1}{2}$$

the probability that the price of the commodity will decrease, P (B) = $\frac{1}{3}$

The probability that the price of the commodity will remain constant = P (c) =?

Since, A, B and C are mutually exclusive events then

$$P(A) + P(B) + P(C) = 1$$

or, $\frac{1}{2} + \frac{1}{3} + P(C) = 1$

$$P(C) = 1 - \frac{1}{2} - \frac{1}{3} = \frac{6-3-2}{6} = \frac{1}{6}$$

Hence, the probability that the price of the commodity will remain constant is $\frac{1}{6}$.

Example 10

A coin is tossed three times. Find the probability of getting

(i) Exactly 2 heads. (ii) At least one head.

(iii) At most 1 head.

Solution:

Here, a coin is tossed three times. So, the sample space (S) is given as

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

The number of heads obtains with their corresponding probability is given below

No. Heads	No. of Cases	Probability
0	1	1/8
1	3	3/8
2	3	3/8
3	1	1/8

Hence,

i) Probability of getting exactly two heads. $= \frac{3}{8}$

ii) Probability of getting at least one head

$$= P(1) + P(2) + P(3)$$
$$= \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$$

iii) Probability of getting at most one head

$$= P(0) + P(1)$$
$$= \frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$$

Example 11

What is the chance that a leap year selected at random will contain 53 Sundays?

Solution:

A leap year contains 366 days. So, there are 52 complete weeks and 2 extra days. Following are the possible combinations for two extra days.

i) Sunday and Monday
ii) Monday and Tuesday
iii) Tuesday and Wednesday
iv) Wednesday and Thursday
v) Thursday and Friday
vi) Friday and Saturday
and vii) Saturday and Sunday.

To have 53 Sundays in a leap year, one of the two extra days must be Sunday. Out of these 7 possibilities, 2 (viz vi and vii) are favourable.

Hence, the probability of 53 Sundays in a leap year $= \frac{2}{7}$

Example 12

A problem in statistics is given for three students A, B and C Whose chances of solving are $\frac{1}{2}, \frac{1}{3}$ *and* $\frac{1}{4}$ respectively. What is the probability that the problem will be solved if all of them then try independently?

Solution:

Here,

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$

$P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$, $(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$ and $(\bar{C}) = 1 - \frac{1}{4} = \frac{3}{4}$

The problem will be solved if at least one can solve the problem.

i.e. $P(A \cup B \cup C) = 1 - (\overline{A \cup B \cup C})$

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$ (By demargan law)

$= 1 - P(\bar{A}).P(\bar{B}).P(\bar{C})$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= \frac{24-6}{24} = \frac{18}{24} = \frac{3}{4}$

Example 13

A bag contains 8 white and 3 black balls. Two balls are drawn one after another. Find the probability of drawing one white and one black ball in any order.

i. If the first ball is replaced.

ii. If the first ball is not replaced.

Solution:

Total number of balls = 8 + 3 = 11

i. With replacement:

Let, W = event of drawing white ball.

B = event of drawing the black ball.

P (one white ball and one black ball)

= P (first white ball and second black ball) + P (first black ball and second white ball)

$= P(W_1 \cap B_2) + P(B_1 \cap W_2)$

$= P(W_1).\ P(B_2) + P(B_1).\ P(W_2)$

$= \frac{8}{11} \times \frac{3}{11} + \frac{3}{11} \times \frac{8}{11}$

$= \frac{24}{121} + \frac{24}{121} = \frac{48}{121}$

= 0.3966

ii. Without replacement

P (one white and one block ball

= P (first white and second black) + P (first black and second white)

$= P(W_1 \cap B_2) + P(B_1 \cap W_2)$

$= P(W_1)\ P(B_2/W_1) + P(B_1)\ P(W_2/B_1)$

$= \frac{8}{11} \times \frac{3}{10} + \frac{3}{11} \times \frac{8}{10}$

$= \frac{48}{110} = \frac{24}{55}$

= 0.4363

Example 14

In a certain assembly plane, three machines A_1, A_2 and A_3 make 30%,45% and 25% respectively of the products. It is known that inexperience 2%, 3% and 2% of the products made by each machine respectively are defective. A finished product is randomly selected,

i. What is the probability that it is defective?

ii. If a product were chosen randomly and found defective, what is the probability that it was made by machine A_1?

Solution:

Let E_1, E_2, and E_2 be the events of production of machines A_1, A_2 and A_3 respectively.

If D be the events of production of defective items, then

$P(E_1) = 30\% = 0.3$,

$P(E_2) = 45\% = 0.45$

and $p(E_3) = 0.25$

Similarly, $P(D/E_1) = 0.02$,

$P(D/E_2) = 0.03$

and $P(D/E_3) = 0.02$

Calculation table:

Machines	Probability	Conditional Prob.	Product
A	0.3	0.02	0.006
B	0.45	0.03	0.0135
C	0.25	0.02	0.005
Total	0.0245		

Now, $P(D) = \sum P(E_i)(D/E_i)$

$= P(E_1)(D/E_1) + P(E_2)(D/E_2) + P(E_3)(D/E_3)$

$= 0.3 \times 0.02 + 0.45 \times 0.03 + 0.25 \times 0.02 = 0.0245.$

Again, $P(D/E_1) = \frac{P(E_1)(D/E_1)}{\sum P(E_i)(D/E_i)}$

$= \frac{0.3 \times 0.02}{0.0245}$

$= 0.2448$

Example 15

Suppose a B.Sc. class contains 60 boys and 40 girls' students. Among the students, 8% boys and 4% girls got a free ship. A student is selected at random from the class and if the free ship is received by the student, what is the probability that the selected student is

ii. a boy?

ii. a girl?

Solution:

Let Band Gare the events that the selected of boys and girls respectively, then

$P(B) = \frac{60}{100} = 0.6, \quad P(G) = \frac{40}{100} = 0.4$

Also, F is the selected student who received the free ship. Then

P (F/B) = Probability that a boy gets free ship = 0.08

P (F/G) = probability that girl gets free ship = 0.04

From Bayes theorem, the probability of boy got free ship is given as,

$$P(B/F) = \frac{p(B).P(F/B)}{P(B).P(F/B) + P(G).P(F/G)}$$

$$= \frac{0.6 \times 0.08}{0.6 \times 0.08 + .4 \times 0.04} = \frac{0.048}{0.065} = \frac{3}{4}$$

Example 16

A man speaks the truth 3 out of 4 times He throws a die and reports that it is six. Find the probability that it is exactly a six.

Solution:

Here, P (six occurs) = $\frac{1}{6} = P(E_1)$,

P (not six occurs) = $\frac{5}{6} = P(E_2)$

A be an event that man reports six, then

P (man speaks truth) = P (A/E_1) = $\frac{3}{4}$

and P (man does not speaks truth) = P (A/E_2) = $1 - \frac{3}{4} = \frac{1}{4}$

$$P(E_1/A) = \frac{P(E_1).\ P(A/E_1)}{P(E_1).\ P(A/E_1) + P(E_2).\ P(A/E_2)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{8}$$

Example 18

A salesperson has a 65 percent chance of making a sale to a customer. The behavior of each successive customer is independent. If three customers A, B and C enter together, what is the probability that the salesperson will make a sale to at least one of the customers?

Solution:

Here,

P(A) = Probability of making a sale to customer A = 0.65.

P(B) = Probability of making a sale to customer B = 0.65

P(C) = Probability of making a sale to customer C = 0.65

Now, P ($\bar{A}$) = 1 - P(A) = 0.35

P ($\bar{B}$) =1 - P(B) = 0.35

P ($\bar{C}$) = 1 - P(C) = 0.35

The probability that the salesperson will make a sale to at least one of the customers is

P (A or B or C)

$= 1 - P(\bar{A} \text{ and } \bar{B} \text{ and } \bar{C})$, since the events are independents.

$= 1 - P(A) \times P(B) \times P(C)$

$= 1 - 0.35 \times 0.35 \times 0.35$

$= 1 - 0.042875 = 0.957125 = 0.9571$

Example 19

There are three machines P, Q and R producing 20%, 50% and 30% articles per hour respectively. These machines are known to be producing 1%, 3% and 2% defective items respectively. One article is selected at random from an hour production of three machines and found to be defective. What is the probability that the defective article is produced from (i) Machine P (ii) Machine Q?

Solution

Here,

P (P) = 20% = 0.2, P (Q) = 50% = 0.5, and P (R) = 30% = 0.3,

Let, E be the event of the defective article.

Now,

P(E/P) = Probability that defective article from machine P =1% = 0.01

P(E/Q) = Probability that defective article from machine Q = 3% = 0.03

P(E/R) = Probability that defective article from machine R = 2% = 0.02

If an article is selected, then

I. It is the product of machine, P= P(P/E)

$$= \frac{P(P).P(E/P)}{P(P).P(E/P) + P(Q)P.(E/Q) + P(R).P(E/R)}$$

$$= \frac{0.2 \times 0.01}{0.2 \times 0.01 + 0.5 \times 0.03 + 0.3 \times 0.02} = \frac{0.002}{0.023}$$

$\therefore P(P/E) = 0.087$

II. It is the product of machine, $Q = P(Q/E)$

$$= \frac{P(Q).P(E/Q)}{P(P).P(E/P) + P(Q)P.(E/Q) + P(R).P(E/R)}$$

$$= \frac{0.5 \times 0.03}{0.2 \times 0.01 + 0.5 \times 0.03 + 0.3 \times 0.02} = \frac{0.015}{0.023} = 0.652$$

$\therefore P(A/E) = 0.652$

Example 20

There are three machines A, B and C producing 1000, 2000 and 3000 articles per hour respectively. These machines are known to be producing 10, 40 and 90 defective items respectively. One article is selected at random from an hour production of three machines and found to be defective. What is the probability that the defective article is produced from

(i) machine A (ii) machine B and (iii) machine C?

Solution:

Here,

$P(A)$ = Probability that selected article from machine A

$$= \frac{1000}{1000+200+3000} = \frac{1}{6}$$

$P(B)$ = Probability that selected article from machine B

$$= \frac{2000}{1000+200+3000} = \frac{2}{6}$$

$P(C)$ = Probability that selected article from machine C

$$= \frac{3000}{1000+200+3000} = \frac{3}{6}$$

Let, E be the event of the defective article.

Now, $P(E/A)$ = Probability' that defective article from machine A

$$= \frac{10}{1000} = 0.01$$

$P(E/B)$ = Probability that defective article from machine B

$$= \frac{40}{2000} = 0.02$$

P(E/C) = Probability that defective article from C = B = $\frac{90}{3000}$ = 0.03

(i) The probability that the defective article is produced from machine A is

$$P(A/E) = \frac{P(A).P(E/A)}{P(A).P(E/A)+P(B)P.(E/B)+P(C).P(E/C)}$$

$$= \frac{\frac{1}{6}\times 0.01}{\frac{1}{6}\times 0.01+\frac{2}{6}\times 0.02+\frac{3}{6}\times 0.03} = \frac{1}{14}$$

$\therefore$ P(A/E) = 0.0714

(ii) The probability that the defective article is produced from machine B is

$$P(B/E) = \frac{P(B).P(E/B)}{P(A).P(E/A)+P(B)P.(E/B)+P(C).P(E/C)}$$

$$= \frac{\frac{2}{6}\times 0.02}{\frac{1}{6}\times 0.01+\frac{2}{6}\times 0.02+\frac{3}{6}\times 0.03} = \frac{4}{14}$$

P(B/E) = 0.2857

(iii) The probability that the defective article is produced from machine C is

P(C/E) =1- P(A/E) P(B/E)

=1 - 0.0714 - 0.2857

= 0.6429

Example 1

Two hundred employees, in a manufacturing concern, havethe following information.

Employee	Skill Level		Total
	High	Low	100
Female	45	55	100
Male	65	35	200
Total	110	90	

If a person selected randomly is found to be a highly skilled one, what is the probability that - a person is (i) male (ii) female.

Solution:

Let's define the events:

A: event that male employee

A: event that female employee

B: event that high skill level employee

B: event that low skill level employee Given information:

Employees	Skill level		Total
	High (B)	Low (B)	
Female (A)	45	55	100
Male (A)	65	35	100
Total	110	90	200

i. If a person selected randomly is a highly skilled one, the probability that the person is male is

$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{65}{110} = 0.5909$$

ii. If a person selected randomly is a highly skilled one, the probability that the person is female is

$$P(\bar{A}/B) = \frac{P(\bar{A} \text{ and } B)}{P(B)} = \frac{45}{110} = 0.4091$$

Alternatively,

$$P(A/B) -1 - P(A/B) = 1 - 0.5909 - 0.4091$$

Example 2

A factory has 3 units A, B and C. A produces 25% of its products, units B produced 25% and units C produces 50%. If the percentage of defective items produced by three units A, B and C are respectively 1%, 2% and 3%. An item is selected randomly from the total production of the factory and found to be defective. What is the probability that it is produced by unit C?

Solution

Here,

P(A) = Probability that selected item from unit A = 25% = 0.25

P(B) = Probability that selected item from unit B = 25% = 0.25

P(C) = Probability that-selected item from unit C = 50% = 0.50

Let D be the event of defective item.

Now,

P(D/A) = Probability that defective item from unit A =1%. = 0.01

P(D/B)=Probability that defective item from unit B =2% = 0,02

P(D/C)= Probability that defective item from unit C = 3% = 0.03

If an item selected random from the total production of the factory is found to bedefective, the probability that it was produced by the unit C is

$$P(C/D) = \frac{P(c).P(D/C)}{P(A).P(D/A) + P(B).P(D/B) + P(c).P(D/C)}$$

$$= \frac{0.50\times0.03}{0.25\times0.01+0.25\times0.02+0.50\times0.03}$$

$$= 0.6667= 66.67\%$$

Example 3

The probability that a management trainee will remain with a company is 0.60. The probability that an employee earns more than Rs. 10,000 per year is 0.50. The probability that an employee is a management trainee who remained with the company or who earns more than Rs 10,000 per year is 0.70. What is the probability that an employee earns more than Rs 10,000 per year given that he is a management trainee who stayed with the company?

Solution

Let, A = Event that management trainee will remain with a company

B = Event that employee earns more than Rs. 10,000 per year.

Here, P(A) = 0.6, P(B) = 0.05, P (A and B) = 0.7 and P(B/A) =?

We have,

P (A or B) = P(A) + P(B) – P (A and B)

or, 0.70 = 0.60 + 0.50 – P (A and B)

P (A and B) = 0.40

The probability that an employee earns more than Rs. 10,000 per year given that he is a management trainee who stay with the company is given as

$$P(B/A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.4}{0.6}$$

P(B/A) = 0.67

Example 4

In a company 10. men are graduates out of 25. If 5 mean are picked out at random, what is the probability that (i) they are all graduates (ii) there are no graduates and (iii) at least one graduate?

Solution

Here;

Number of men in a company = 25

If 5 men are picked out at random,

The exhaustive number of cases (n)= $^{25}C_5$ = 53130

i. $$P(\text{all 5 are graduate}) = \frac{m}{n} = \frac{10C5}{53130} = 0.0047$$

$$P(\text{no graduate}) = \frac{m}{n} = \frac{C(15, 5)}{53130} = 0.0565$$

P (at least one graduates) = 1 – P (no graduates)

= 1 – 0.0565

= 0.9435

Example 5

A problem in statistical methods is given to the three students A, B and C whose probability of solving it is 1/2, 3/4 and 1/4 respectively. What is the probability that the problem (I) Will be solved? (ii) Will not be solved if they try independently?

Solution

Here,

(A) = Probability that the problem will be solved by the student A = $\frac{1}{2}$

P(B) = Probability that the problem will be solved by the student B = $\frac{3}{4}$

P(C) = Probability that the problem will be solved by the student C = $\frac{1}{4}$

Now, $P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

P(B)=1-P(B)= 1 - P(C) = $1 - \frac{3}{4} = \frac{1}{4}$

i. The probability that the problem will be solved is

P (A or B or C) $= 1 - P(\bar{A} \text{ and } \bar{B} \text{ and } \bar{C})$

=1 - P(A). P(B). P(C)

$= 1 - \frac{1}{2} . \frac{3}{4} . \frac{1}{4} = \frac{29}{32} = 0.90625$

ii. The, probability that the problem will not be solved is

= 1- P (the problem is solved)

= 1- 0.90625

= 0.09375

Example 6

A bag contains 6 black, 4 white and 8 red balls. If three balls are drawn at random, find the probability that (i) all the 3 balls are black (ii) all the 3 are red and (iii) 2 are white and 1 is black

Solution

Here, Total number of balls = 6 black + 4 white + 8 red =18

If three balls are drawn at random, exhaustive number of cases, n $^{18}C_3$ = 816 ways

i. Favorable number of cases for getting 3 black balls out of 6 black balls, m = $^{6}C_3$ = 20 ways

P (all the 3 balls are black) = $\frac{m}{n} = \frac{20}{816} = 0.0245$

ii. Favorable number of cases for getting 3 red balls 8 red balls

= $^{8}C_3$ = 56 ways

P (3 red balls) = $\frac{m}{n} = \frac{56}{816} = 0.068$

iii. Favorable number of cases for getting 2 white and 1 black ball,

m = number of selection 2 white balls out of 4 white balls and 1 black ball out of 6 black balls.

= $^{4}C_2 \times {}^{6}C_1 = 6 \times 6 = 36$ ways

P (2 white and 1 black ball) = $\frac{m}{n} = \frac{36}{816} = 0.0441$

Example 7

In an examination of MBA level, 40% failed in Accountancy, 25% failed in statistics and10% failed in both Accountancy and statistics. A student is selected at random. What is the probability that the selected student has failed in (a)statistics or Accountancy? (b) statistics are given that he has failed in Accountancy? (c) d in accountancy given that he has failed in statistics?

Solution

Let, A = event that student has failed in accountancy.

B = event that student has failed in statistics

P(A) = 40% = 0.40, P(B) = 25% = 0.25 and P (A and B) =10% = 0.10

A student is selected at random.

(a) The probability that the selected student has failed in statistics or accountancy is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= 0.40 + 0.25 - 0.10$$

$$= 0.55 = 55\ \%.$$

(b) The probability that the selected student has failed in statistics given that he has failed in accountancy is

$$P(B/A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.1}{0.4} = 0.25 = 25\ \%$$

(c) The probability that the selected student has failed in accountancy given that he has failed in statistics is

$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.1}{0.25} = 0.40 = 40\ \%$$

1.8 Points to Remember

1. **Sample Space**: All possible outcomes (e. g. flip a coin: H, T)
2. **Outcome**: Result of an experiment. (e. g. flip a coin: H)
3. **Probability**: Chance that an event will occur which lies between 0 and 1

   0 → not going to happen.

   1 → certain to happen

   0.5 → equally likely to happen or not happen
4. **Theoretical Probability**: What should happen.

   $P(E) = \frac{\text{Number of outcomes}}{\text{Sample space}}$ e.g., throw a die and want 6 = $P(6) = \frac{1}{6}$
5. **Empirical Probability**: What happens in an experiment

   e.g., flip a coin 500 times and get 257 heads, $P(H) = \frac{257}{500} = 0.514$
6. **Law of large number**: More trials are done, the closer the empirical probability is to the theoretical probability.
7. **Conditional probability**: Probability of B given A has already occurred and given as, P (B/A)

   $P(B/A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{P(A \cap B)}{P(A)}$

   $P(A \cap B) = P(A \text{ and } B)$ = Probability both occur together = P(A). P(B/A)

   P (A or B) = Probability of A or B or both = $P(A) + P(B) - P(A \cap B)$
8. **Independent Event**: One occurring does not change the probability of the other occurring.

   P(A/B) = P(A)

   P(B/A) = P(B)

   P (A and B) = P(A). P(B)
9. **Mutually Exclusives:** Both cannot occur at the same time.

   P (A and B) = 0
10. **Law of Total Probability**: Let, $\{E_1$ and $E_2\}$ are two mutually exclusive and exhaustive events and have a non-zero probability of occurrence and A be an event, then

    $P(A) = P(E_1).\ P(A / E_1) = P(E_2).\ P(A / E_2)$