Chapter 3: Sampling and Estimation

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Example 9

A random sample of 40 students is drawn from a certain school and their height showed a mean of 68.6 inches and standard deviation of 2.5 inches. Construct 95% and 98 % confidence intervals for the mean height of all students of the school.

Solution:

Let, μ - Population mean (mean height of all)

Sample mean $(\bar{x}) = 68.6$

St. dev. $(\sigma) = 2.5$

No. of sample, n = 40 >30

Confidence interval $(1 - \alpha) = 0.95$

$\Rightarrow \alpha = 0.05$

The confidence interval at 95% confidence is given by

$$\bar{x} \pm Z_{0.025} \, \sigma / \sqrt{n}$$

$$= \left(68.6 \pm 1.96 \frac{2.5}{\sqrt{40}} \right) \qquad \because Z_{0.055} = 1.96$$

$$= (67.726, 69.274)$$

Again, for 98% confidence, the confidence interval is given by

$$\left(\bar{x} \pm Z_{0.01} \, \sigma / \sqrt{n} \right)$$

$$= \left(68.6 \pm 2.33 \frac{2.5}{\sqrt{40}} \right)$$

$$= (67.678, 69.521)$$

Example 12

From a sample of 14 objects, it is found that the mean and standard deviation were 17.85 and 1.955 respectively. Find 95% fiducial limits for mean.

Solution:

Here,

$n = 14, 1 - \alpha = 95\%$ and $\alpha = 5\%$

Hence, the confidence interval is

$$\left(\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n-1}}\right) = \left(17.8 \pm 2.16 \frac{1.955}{\sqrt{13}}\right) = (16.68, 19.02)$$

Example 13

A research worker wants to determine the average time it takes machine to rotate the tires of a car and she wants to be able to ass with 95% confidence that the mean of her sample is off by at m 0.05 minutes if she can presume from experience that σ 1.6 minutes. How long sample will she has taken?

Solution:

Here, $\sigma = 1.6,\ E = 0.05,\ 1 - \alpha = 0.95$ *and* $n = ?$

The sample size is given by

$$n = \left(Z_{\alpha/2} \frac{\sigma}{E}\right)^2$$

$$= \left(1.96 \frac{1.6}{0.05}\right)^2, \because Z_{0.025} = 1.96$$

$$= 39.3 = 40$$

Example 14

The result X of a stress test is taken to be a normally distributed random variable with mean μ and s. d 1.3. It is required to have a 95% confidence interval for/with total width less than 2. Find the least number of tests that should be carried out.

Solution:

Here,

$X \sim N(\mu, 1.3^2)$,

$1 - \alpha = 95\%$

$Z_{\alpha/2} = 1.96$

So, the confidence interval for μ is $\left(\bar{x} \pm Z_{\alpha/2} \sigma/\sqrt{n}\right)$

And the width of the confidence interval is

$$\left(2 \cdot Z_{\alpha/2} \sigma/\sqrt{n}\right) = 2(1.96)\frac{(1.3)}{\sqrt{n}}$$

From question

$$2 \times 1.96 \frac{1.3}{\sqrt{n}} < 2$$

$\therefore$ n= 6.49~ 7.

So, the least number of tests is 7.

Example 16

In 40 tosses of a coin, 24 heads were obtained. Find 95% confidence interval for proportion of heads.

Solution:

Here, (n) = 40 and

no. of heads appeared = 24

The sample proportion of head (p) = $\frac{24}{40}$ = 0.6

and q=1-p=1-0.6=0.4

$1-\alpha$ = 95% i.e., α = 0.05 and $Z_{\alpha/2} = Z_{0.025} = 1.96$

Hence, the confidence interval (C.I.) is given as

$$= p \pm Z_{\alpha/2}\sqrt{\frac{pq}{n}}$$

$$= 0.6 \pm 1.96\sqrt{\frac{0.6\times0.4}{40}}$$

$$= (0.448, 0.752)$$

Example 17

A random sample of 80 people from a community of 300 should that 30 were a smoker. Find 99% confidence intervals for the proportion of smokers.

Solution:

Here,

Population size (N)=300 (finite)

Sample size(n) =80

No. of smoker(X) =30

Sample proportion $(p) = \frac{X}{n} = \frac{30}{80} = 0.375$

$$\therefore q = 1 - p = 1 - 0375 = 0.625$$

Confidence coefficient $(1-\alpha) = 99\ \%$

$$\therefore \alpha = 0.01$$

and $Z_{\alpha/2} = Z_{0.005} = 2.57$

So, the confidence interval is given by

$$= p \pm Z_{\alpha/2}\sqrt{\frac{pq(N-n)}{n(N-1)}}$$

$$= 0.375 \pm 2.57\sqrt{\frac{PQ(300-80)}{80(300-1)}}$$

$$= 0.375 \pm 2.57\sqrt{2.155}$$

$$= (0.375 \pm 0.119)$$

$$= (0.2556, 0.495)$$

Determining the sample size for estimating a population proportion

To find the sample size for estimating population proportion (P)

We have, $$Z_{\alpha/2}\sqrt{\frac{pq}{n}} = E$$

$$n = \frac{Z^2_{\alpha/2}\, pq}{E^2}$$

Where, E is the error of estimation i.e., the difference between the estimated wants to estimate and the true value of the proportion

Example 18

A market analyst wants to estimate the proportion of shoppers who will buy a new type of liquid detergent. How large a sample should be taken to estimates that proportion differs by 0.04 with 95% confidence?

Solution:

Here,

We have to estimate the parameter i.e., population probability (P).

Error (E) = 0.04

Confidence coefficient = $1 - \sigma$ = 95%

$\therefore \sigma = 0.05$ and $Z_{\alpha/2} = Z_{0.025} = 1.96$

Here, the value of P and q are not known, the best way of proceeding is to select the value of p and q that produce the largest possible value of n. Thus, we set up p = q = 0.5. Hence the sample size n is given by

$$n = \left(Z_{\alpha/2}\frac{\sigma}{E}\right)^2$$

In the case of proportion, $n = \frac{Z_\alpha{}^2 P\,Q}{E^2}$

$$= \frac{1.96^2 \times 0.5 \times 0.5}{(0.04)^2} = 423$$

Example 21

A company selling tooth – paste ABC wishes to estimate the proportion of people who prefer their brand ABC. It wishes to keep the error within 2 percent, with a risk of 0.0456. How large a sample must be taken ? Given that the proportion of people prefer their brand ABC are equally likely.

Solution :

With the usual notation, we have,

σ = risk = 0.0456

$$\therefore \frac{\alpha}{2} = \frac{0.0456}{2}$$

$$= 0.0228$$

The value of Z_α at 0.0228 is 2.

E = error = 2% = 0.02, n = ?, $P = Q = \frac{1}{2}$

$$\text{Now, } n = PQ\left(\frac{Z_\alpha}{E}\right)^2 = \frac{1}{2} \times \frac{1}{2}\left(\frac{2}{0.02}\right)^2$$

$$= \frac{1}{4} \times \frac{4}{0.004}$$

$$= 2500$$

Example 1

A zookeeper took a random sample of 50 days and observed how much food an elephant ate on each of those days. The mean was 350 kg and the standard deviation was 25 kg. Construct 90% and 99% confidence intervals for a true population mean.

Solution:

Here,

Sample size, n = 50 days

Sample mean, $\bar{X}$ = 350 kg

Sample standard deviations, s = 25kg

Now, the standard error of sample mean is

S.E. $(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}}$, [For large sample, $\hat{\sigma} = s$]

$= \frac{25}{\sqrt{50}} = 3.54$

For 90% confidence level:

Level of significance, α = 10% = 0.10

$\therefore Z_\alpha = Z_{0.10} = 1.645$

90% confidence interval for true population mean is

$\bar{X} \pm Z_\alpha$ S.E. $(\bar{X})$ = 350 ± 1.645 x 3.54= 350 ± 5.8233

Lower confidence limit = 350 - 5.8233 = 344.18 kg

Upper confidence limit = 350 + 5.8283 = 355.82 kg

This indicates that P(344.18 < μ < 355.82) = 0.90, on the basis of 50 samples.

For 99% confidence level:

Level of significance, α = 1% = 0.01

$\therefore Z_\alpha = Z_{0.01} = 2.575$

99% confidence interval for true population mean is

$\bar{X} \pm Z_\alpha$ S.E. $(\bar{X})$ = 350 ± 2.575 x 3.54 = 350 ± 9.1155

Lower confidence limit =350 - 9.1155= 340.88 kg

Upper confidence limit = 350 + 9.115...

This indicates that P (340.88 < μ < 359.12), on the basis of 50 samples.

Example 2

The quality control manager at a light bulb factory needs to estimate the mean life of a large shipment of life bulbs. The process standard deviation is known to be 100 hours. A random sample of 64 light bulbs indicated a sample mean life of 350 hours.

i) Obtain the standard error of the mean.
ii) Set up a 95% confidence interval estimate of the true population mean of light bulbs in -shipment.

Solution:

Here,

Sample size, n = 64

Sample mean, $\bar{X}$ = 350 hrs.

Population standard deviations, σ = 100

a. $S.E.(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{100}{\sqrt{64}} = 12.5$

b. For 95% confidence level:

Level of significance, $\alpha = 5\% = 0.05$

$\therefore Z_{\alpha} = Z_{0.05} = 1.96$

95% confidence interval for the true population mean life of light bulbs is

$$\bar{X} \pm Z_{\alpha} S.E.(\bar{X}) = 350 \pm 1.96 \times 12.5 = 350 \pm 24.5$$
$$= (350 - 24.5, 350 + 24.5)$$
$$= (325.5, 374.5)$$

Hence, we conclude with 95% confidence that the mean life of light bulbs lies between 325.5 hours to374.5 hours based on 64 light bulbs.

Example 3

A researcher got involvement in knowing the responsible attitude of the people in a certain locality. He estimates the standard deviation is 0.05 sec. How large a sample must he take to be 95% confident that the error of his estimate of the mean will not exceed 0.01 sec?

Solution:

Here,

Sample size (n) = 400 mobile sets

Number of damaged mobile sets (X) = 20

Standard deviation, a = 0.05seconds

Error, E = 0.01 seconds

Confidence level, 1 - a = 95%

Level of significance, a = 5% = 0.0

$Z_{\alpha} = Z_{0.05} = 1.96$

Sample size, n =?

We have,

$$n = \left(\frac{\sigma Z_{\alpha}}{E}\right)^2$$

$$= \left(\frac{0.05 \times 1.96}{0.01}\right)^2$$

$$= 96.04$$

$$= 96$$

Hence, the required sample size is at least 96.

Example 4

A random sample of 600 apples was taken from a large consignment and 75 of them were found to be bad. Find the 99 percent confidence limits for the percentage of bad apples.

Solution :

Here,

Sample size, n =600

Defective items, X = 75

Proportion of bad items, $p = \frac{X}{n} = \frac{75}{600} = 0.125$

$q = 1 - p = 1 - 0.125 = 0.875$

Confidence level = 99%.

Level of significance, α = 1% = 0:01

$\therefore \quad Z_\alpha = Z_{0.01} = 2.575$

99% confidence limit for percentage of bad apples is

$$P \pm Z_{0.01}\sqrt{\frac{pq}{n}} = 0.125 \pm 2.575\sqrt{\frac{0.125 \times 0.875}{600}}$$

$$= 0.125 \pm 0.0348$$

Lower confidence limit = 0.125 - 0.0348 = 0.0902 = 9.02%

Upper confidence limit = 0.125 + 0.0348 = 0.1598 =15.98%

Hence, we conclude with 99% confidence that 9.02% to 15.98% of apples are bad inthe large consignment, on the basis of 600 apples.

Example 5

A sample of 400 mobile sets was taken from a lot. 20 mobile sets were found to be damaged.

a. Find the standard error of the proportion of damaged mobile sets.

b. Estimate 95% confidence interval of the percentage of damaged mobile set.

Solution:

Sample size, n = 400

Number of damaged items, X = 20

The proportion of damaged items, $p = \frac{X}{n} = \frac{20}{400} = 0.05$

$Q = 1 - p = 1 - 0.05 = 0.95$

a. The standard error of proportion of damaged items is given as

$S.E.(p) = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{pq}{n}}$, [for a large sample, $\hat{P}$ = p and $\hat{Q}$ = q]

$= \sqrt{\frac{0.05 \times 0.95}{400}}$

$= 0.0109$

b. For 95% confidence level:

Level of significance, α = 5% = 0.05

$\therefore \quad Z_\alpha = Z_{0.05} = 1.96$

95% confidence interval of percentage of damaged mobile sets is

$p \pm Z_\alpha S.E.(p) = 0.05 \pm 1.96 \times 0.0109$

$= 0.05 \pm 0.0214$

$= (0.05 - 0.0214, 0.05 + 0.0214)$

$= (0.0286, 0.0714)$

= (2.86%, 7.14%)

i. e., 2.86% < P < 7.14% with 95% confidence.

Hence, we conclude with 95% confidence that 2.86% to 7.14% of mobile sets are damaged in the lot.

Example 6

A sample of 500 bulbs of a company showed an average lifetime of 1400 hours with the standard deviation of 30 hours. Obtain 95% and 99% confidence limits for the population mean.

Solution:

Sample size, n = 500

Sample mean, $\bar{X} = 1400$

Sample standard deviations, s = 30 hrs.

Now, the standard error of sample mean is

$S.E.(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}}$, [For large sample, $\hat{\sigma} = s$]

$$= \frac{30}{\sqrt{500}} = 1.3416$$

For 95% confidence level:

Level of significance, $\alpha = 5\% = 0.05$

$\therefore Z_\alpha = Z_{0.05} = 1.96$

95% confidence interval for true population mean is

$\bar{X} \pm Z_\alpha \text{S.E.} (\bar{X}) = 1400 \pm 1.96 \times 1.3416$

$= 1400 \pm 2.6295$

Lower confidence limit = 1400 - 2.6295 = 1397.37

Upper confidence limit =1400 + 2.6295 = 1402.63

This indicates that P (1397.37 < μ < 1402.63) = 0.90, on the basis of 50 samples.

For 99% confidence level:

Level of significance, $\alpha = 1\% = 0.01$

$\therefore Z_\alpha = Z_{0.01} = 2.575$

99% confidence interval for true population mean is

$\bar{X} \pm Z_\alpha \text{ S.E. } (\bar{X}) = 1400 \pm 2.575 \times 1.3416 = 1400 \pm 3.4546$

Lower confidence limit = 1400 - 3.4546 = 1396.5456

Upper confidence limit = 1400 + 3.4546 = 1403.45

This indicates that P (1396.55 < μ < 1403.45) = 0.99, on the basis of 500 sample light bulbs.

Example 7

A random sample of 700 units from a large consignment showed that 147 were damaged. Find 95% confidence limits for the proportion of damaged units in the consignment.

Solution:

Here, Sample size, n = 700 Number of damaged units, X = 147

The sample proportion of damaged units, $p = \frac{X}{n} = \frac{147}{700} = 0.21$

$q = 1 - p = 1 - 0.21 = 0.79$

Confidence level, $1 - \alpha = 95\%$

Level of significances, $\alpha = 5\% = 0.05$

Now, 95% confidence limits for the proportion of damaged units in the consignment is

$$p \pm Z_{\alpha} S.E.(p) = P \pm Z_{0.05}\sqrt{\frac{pq}{n}} = 0.21 \pm \sqrt{\frac{0.21 \times 0.79}{700}}$$

Lower confidence limit = 0.21 - 0.0302 = 0.1798 = 17.98%

Upper confidence limit = 0.21 + 0.0302 = 0.2402 = 24.02%

Hence, we conclude with 95% confidence that 17.98% to 24.02% are damaged units in the consignment, on the basis of 700 units.

Example 8

The quality control manager at a light bulb factory needs to estimate the mean life of a large shipment of life bulbs. The process standard deviation is known to be 100 hours. A random sample of 64 light bulbs indicated the sample mean life of 350 hours.

a) Obtain the standard error of the mean.
b) Set up a 95% confidence interval estimate of the true population mean of light bulbs in shipment.
c) Do you think that the manufacture has the right to state that the bulb life of 400 hours?

Solution:

Here,

Sample size, n = 64

Sample mean, $\bar{X}$ = 350 hrs.

Population standard deviations, $\sigma = 100$

a. $S.E.(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{100}{\sqrt{64}} = 12.5$

b. For 95% confidence level:

Level of significance, $\alpha = 5\% = 0.05$

$\therefore\ Z_\alpha = Z_{0.05} = 1.96$

95% confidence interval for the true population mean life of light bulbs is

$$\bar{X} \pm Z_\alpha \text{S.E.}(\bar{X}) = 350 \pm 1.96 \times 12.5$$
$$= 350 \pm 24.5$$
$$= (350 - 24.5, 350 + 24.5)$$
$$= (325.5, 374.5)$$

Hence, we conclude with 95% confidence that the mean life of light bulbs lies between 325.5 hours to 374.5 hours based on 64 light bulbs.

c. Null hypothesis $H_0: \mu = 400$, i. e. the average life of the bulbs is 400

Alternative hypothesis $H_0: \mu \neq 400$, i. e. the average life of the bulbs is not 400 (two-tailed test).

Decision: Since the population average life of 400 hours does not include in 95% confidence interval estimate of the population mean and we reject H_0. So, the manufacturer does not have the right to state that the average life of light bulbs is not 400 hours.

Example 9

If the population proportion of success is 0.65 and n = 10, what sampling error when the acceptance region is 0.95?

Solution:

Population proportion of success, P = 0.65

Q = 1 – P = 0.35

Sample size, n =10

Acceptance region, (i.e., confidence level), $1 - \alpha = 0.95$

Level of significance, a = 0.05

$Z_\alpha = Z_{0.05} = 1.96$

Sampling error, $E = |p - P| = ?$

We have,

$$n = \frac{Z_\alpha^2 PQ}{E^2}$$

or, $E^2 = \frac{Z_\alpha{}^2 PQ}{n} = \frac{1.96^2 0.65 \times 0.35}{10} = 0.0874$

$E = 0.2956$

The sampling error is 0.2956.

Example 12

A researcher wants to estimate the mean of the population by using a sufficiently large sample. The probability is 0.90 that the sample mean will not differ from the actual mean by more than 10% of the population standard deviation. How large should be the sample?

Solution

Here, Confidence level, $1 - \alpha = 0.90$

Level of significance, $\alpha = 0.10$

$Z_\alpha = 1.645$

The margin of error, $E = |\bar{x} - \mu| =$ 10% of population s. d. $= 0.10.\sigma$

Sample size; n =?

We have,

$n = \left(\frac{Z_\alpha .\sigma}{E}\right)^2 = n = \left(\frac{1.645 \times \sigma}{0.10 \times \sigma}\right)^2 = 270.6025 \sim 271$

Hence, the required sample size is 271.