Skewness, Kurtosis, and Moments

BBS 1st Year

Skewness, Kurtosis and Moments

→ The lack of symmetry of a distribution is called skewness.

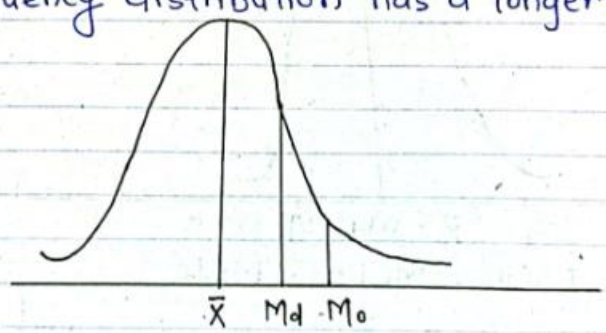
→ It relates to the shape but not the size of a frequency curve.

→ A distribution which is not symmetrical (same) is said to be skewed. So, skewness is the lack of symmetry.

→ Generally, there are two types of skewness.
a. Positive skewness
b. Negative skewness

a. Positive skewness

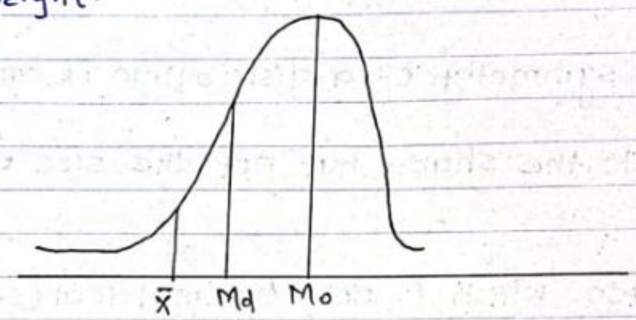
Skewness is said to be positive when the curve of the frequency distribution has a longer tail on the right.



Mean > Median > Mode

b. Negative skewness

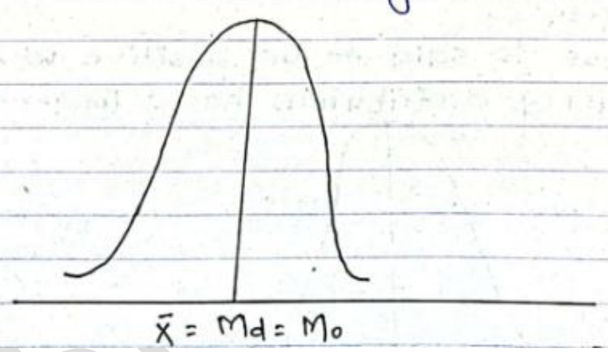
Skewness is said to be negative if the curve of the frequency distribution is elongated more on the left than the right.



Mean < Median < Mode

Zero skewness | symmetrical

If the tails of the curve of the frequency distributi[on] on either side of the central value are equal, then the distribution is said to be symmetrical



* Measure of skewness | coefficient of skewness

a. Karl Pearson's measure of skewness

First measure of skewness

$$S_K(P) = \frac{\text{Mean} - \text{Mode}}{\text{Standard Deviation}} = \frac{\bar{X} - M_o}{\sigma}$$

Second measure of skewness

$$S_K(P) = \frac{3(\bar{X} - M_d)}{\sigma}$$

Decision:

a. If $S_K = 0$, the distribution is symmetrical (normal)
b. If $S_K > 0$, the distribution is positively skewed
c. If $S_K < 0$, the distribution is negatively skewed

b. Bowley's Measure of skewness

The second coefficient of skewness based on quartiles is known as Bowley's coefficient of skewness.

$$S_K(B) = \frac{Q_3 + Q_1 - 2M_d}{Q_3 - Q_1}$$

c. Kelly's coefficient of skewness

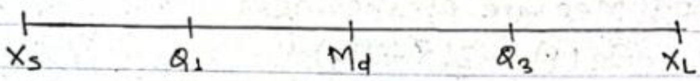
Kelly's coefficient of skewness is based on percentiles or deciles.

$$S_K(\text{Kelly}) = \frac{P_{90} + P_{10} - 2M_d}{P_{90} - P_{10}} = \frac{P_{90} + P_{10} - 2P_{50}}{P_{90} - P_{10}}$$

OR,

$$S_K(\text{Kelly}) = \frac{D_9 + D_1 - 2D_5}{D_9 - D_1}$$

* Five Number Summary



a. Minimum Value(X_S) : The small item in the set of observation.

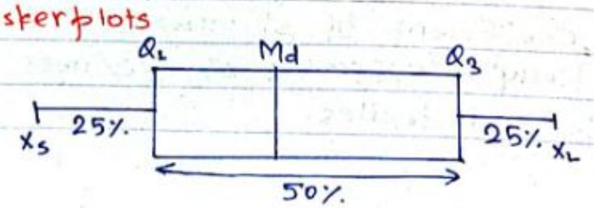
b. Lower Quartile(Q_1) : The value below of which lie 25% of the set of observations.

c. Median (M_d) : The value below and above of which 50% of the observations.

d. Upper Quartile (Q_3) : The value above of which lies 25% of the observations

e. Maximum Value(X_L) : The largest item in the set of observations.

* Box-And-Whisker plots



Moments:

The arithemstic average of the various powers of the deviations of the items in a distribution from their arithmetic mean are known as the moments of the distribution.

Moments about the Mean (central moments)

Individual Series:

$$\mu_r = \frac{\Sigma (X-\bar{X})^r}{n} = \frac{\Sigma x^r}{n}$$

Where,

X = Arithmetic Mean

$x = X - \bar{X}$

n = Number of terms

r = 1, 2, 3, 4 are the first four moments about the mean.

Here,

μ_1 = First moment about Mean $= \frac{\Sigma x}{n} = 0$

μ_2 = Second moment about Mean $= \frac{\Sigma x^2}{n}$

μ_3 = Third moment about Mean $= \frac{\Sigma x^3}{n}$

μ_4 = Fourth moment about Mean $= \frac{\Sigma x^4}{n}$

Discrete and continuous Series:

$$\mu_r = \frac{\Sigma f(X-\bar{X})^r}{N} = \frac{\Sigma f x^r}{N}$$

Where, $x = X - \bar{X}$

N = Total frequency

μ_1 = First moment about Mean $= \frac{\Sigma f x}{N}$

μ_2 = Second moment about Mean $= \frac{\Sigma f x^2}{N}$

μ_3 = Third moment about Mean $= \frac{\Sigma f x^3}{N}$

μ_4 = Forth moment about Mean $= \frac{\Sigma f x^4}{N}$

Where,

X = middle value of class (for continuous series)

Moments about an Arbitary Point (Raw moments)

The calculation of the moments about the mean will be easy only when the arithmetic mean of the given series be in whole number. But if the arithmetic mean be not a whole number, then the calculation will be too difficult. In such cases, we find the moments about any number (arbitrary number). such type of moments are known as raw moments. It is denoted by μ'.

Individual Series:

$$\mu'_r = \frac{\Sigma(x-a)^r}{n} = \frac{\Sigma d^r}{n}$$

where,

n = number of observations

$x - a = d$

μ'_1 = First moment about a $= \frac{\Sigma d}{n}$

μ'_2 = Second moment about a $= \frac{\Sigma d^2}{n}$

μ'_3 = Third moment about a $= \frac{\Sigma d^3}{n}$

μ'_4 = Fourth moment about a $= \frac{\Sigma d^4}{n}$

Discrete Series:

$$\mu'_r = \frac{\Sigma f(x-a)^r}{N} = \frac{\Sigma f d^r}{N}$$

μ'_1 = First moment about a = $\frac{\Sigma fd}{N}$

μ'_2 = Second moment about a = $\frac{\Sigma fd^2}{N}$

μ'_3 = Third moment about a = $\frac{\Sigma fd^3}{N}$

μ'_4 = Fourth moment about a = $\frac{\Sigma fd^4}{N}$

Continuous series:

$$\mu'_r = \frac{\Sigma f(x-a)^r}{N} \times h^r = \frac{\Sigma fd'^r}{N} \times h^r$$

where,

$d' = \frac{x-a}{h}$, h = common factor

μ'_1 = First moment about a = $\frac{\Sigma fd'}{N} \times h$

μ'_2 = Second moment about a = $\frac{\Sigma fd'^2}{N} \times h^2$

μ'_3 = Third moment about a = $\frac{\Sigma fd'^3}{N} \times h^3$

μ'_4 = Fourth moment about $a = \frac{\Sigma f d'^4}{N} \times h^4$

* Relation between central moments and raw moments:

$$\mu_1 = \mu'_1 - \mu'_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'^2_1$$

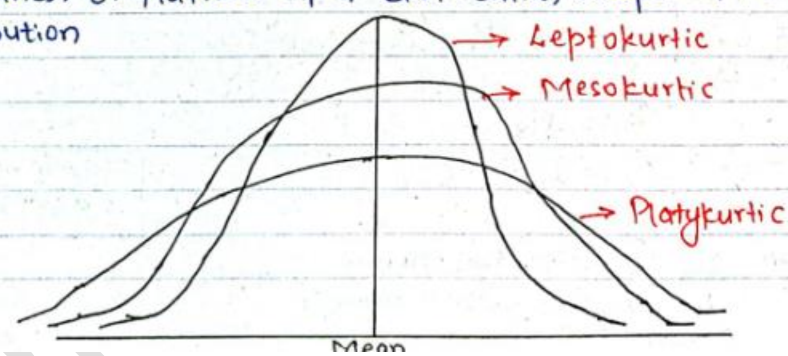
$$\mu_3 = \mu'_3 - 3 \cdot \mu'_2 \cdot \mu'_1 + 2 \mu'^3_1$$

$$\mu_4 = \mu'_4 - 4 \mu'_3 \cdot \mu'_1 + 6 \mu'_2 \cdot \mu'^2_1 - 3 \mu'^4_1$$

Note: μ_2 is also known as Variance $(\mu_2 = \sigma^2)$

* Kurtosis

Besides centeral tendency, dispersion and skewness, the fourth characteristic of the frequeny distribution is the kurtosis. Kur-tosis indicates the peakedness of the distribution. In statistics kurtosis refers to the degree of peakedness or flatness of a distribution compared to a normal distribution



Measure of kurtosis:

Kurtosis is measured by the coefficient of kurtosis. The simple measure of kurtosis based on both quartiles and percentiles is percentile coefficient of kurtosis.

$$K = \frac{\frac{1}{2}(Q_3 - Q_1)}{P_{90} - P_{10}}$$

Where. Q_3 = upper Quartile
Q_1 = Lower Quartile
P_{90} = 90^{th} Percentile
P_{10} = 10^{th} Percentile

Test for kurtosis:

a. When $K = 0.263$, the distribution is mesokurtic (normal)
b. When $K > 0.263$, the distribution is leptokurtic (more peaked)
c. When $K < 0.263$, the distribution is platykurtic (flat topped)

* Coefficient of skewness and kurtosis Based on Moments:

The relative measure of skewness based on moments, denoted by Beta one, β_1, is given by

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

Where. μ_3 = Third central moment
μ_2 = Second central moment

The alternative relative measure of skewness based on moments, denoted by gamma one (γ_1) is given by.

$$\gamma_1 = \sqrt{\beta_1}$$

$$= \sqrt{\frac{\mu_3^2}{\mu_2^3}} = \frac{\mu_3}{\mu_2^{3/2}}$$

Interpretation:

If $\mu_3 = 0$ i.e $\beta_1 = 0$ or $\gamma_1 = 0$, the distribution is symmetrical.

If $\mu_3 > 0$ i.e $\gamma_1 > 0$, the distribution is positively skewed.

If $\mu_3 < 0$ i.e $\gamma_1 < 0$, the distribution is negatively skewed.

The relative measure of kurtosis based on moments is given by the coefficient β_2 or γ_2 defined by Karl Pearson

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \quad , \quad \gamma_2 = \beta_2 - 3$$

Interpretation:

If $\beta_2 = 3$, or $\gamma_2 = 0$, the distribution is mesokurtic.

If $\beta_2 > 3$, or $\gamma_2 > 0$, the distribution is leptokurtic

If $\beta_2 < 3$, or $\gamma_2 < 0$, the distribution is platykurtic.

2076 Q.N.16

Soln

Calculation of Karl-Pearson's Coefficient of skewness

Wages (Rs.)	No. of workers (f)	Mid-value (X)	$d' = \frac{X-85}{h(10)}$	fd'	fd'^2
40-50	10	45	-4	-40	160
50-60	15	55	-3	-45	135
60-70	20	65	-2	-40	80
70-80	28 f_0	75	-1	-28	28
80-90	35 → f_1	A = 85	0	0	0
90-100	25 f_2	95	1	25	25
100-110	18	105	2	36	72
110-120	10	115	3	30	90
120-130	8	125	4	32	128
	$N = \Sigma f = 169$			$\Sigma fd' = -30$	$\Sigma fd'^2 = 718$

We know that,

Karl-Pearson's coefficient of skewness, $S_{K(P)} = \frac{\bar{X} - M_o}{\sigma}$

Calculation of Mean

$$\text{Mean } (\bar{X}) = A + \frac{\Sigma fd'}{N} \times h$$

$$= 85 + \frac{(-30)}{169} \times 10$$

$$= \text{Rs. } 83.22$$

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Calculation of Standard deviation

Standard Deviation (σ) = $\sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times h$

$= \sqrt{\frac{718}{169} - \left(\frac{-30}{169}\right)^2} \times 10$

$= 20.54$

Calculation of Mode

Since the highest frequency is 35, the modal class is 80 - 90. So,

We have,

$L = 80$, $f_1 = 35$, $f_0 = 28$, $f_2 = 25$, $h = 10$

$$\text{Mode } (M_0) = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 80 + \frac{35 - 28}{2 \times 35 - 28 - 25} \times 10$$

$$= 84.12$$

Now,

$$S_k(P) = \frac{\bar{X} - M_0}{\sigma} = \frac{83.22 - 84.12}{20.54} = -0.04$$

Since, $S_k(P) = -0.04 < 0$, The given distribution is negatively skewed.

2061 2nd Q.N.4

Soln

Calculation of quartile values

Daily wages (Rs.)	No. of workers (f)	c.f
Below 100	20	20
100 – 150	81	101
150 – 300	120	221
300 - 500	150	371
500 - 1000	130	501
1000 - 1500	30	531
1500 - 2000	10	541
Above 2000	5	546
	$N = \Sigma f = 546$	

We know that,

Coefficient of skewness based on Quartile value is Bowley's coefficient of skewness. so,

$$S_k(B) = \frac{Q_3 + Q_1 - 2M_d}{Q_3 - Q_1}$$

Calculation of Q_1

$\frac{N}{4} = \frac{546}{4} = 136.5$. The c.f just greater than 136.5 is 221 which lies in class 150-300.

We have,

$L = 150$, $f = 120$, $c.f = 101$, $h = 150$

$$Q_1 = l + \frac{N/4 - c.f}{f} \times h$$

$$= 150 + \frac{136.5 - 101}{120} \times 150$$

$$= 194.375$$

Calculation of Q_3

$\frac{3N}{4} = \frac{3 \times 546}{4} = 409.5$. The c.f just greater than 409.5 is 501 which lies in class 500-1000.

We have.

$l = 500$, $f = 130$, $c.f = 371$, $h = 500$

$$Q_3 = l + \frac{3N/4 - c.f}{f} \times h$$

$$= 500 + \frac{409.5 - 371}{130} \times 500$$

$$= 648.076$$

Calculation of Median (Md)

$\frac{N}{2} = \frac{546}{2} = 273$. The c.f. just greater than 273 is 371 which lies in class 300-500.

We have.

$l = 300$, $f = 150$, $c.f = 221$, $h = 200$

$$Md = L + \frac{N/2 - c.f}{f} \times h$$

$$= 300 + \frac{273 - 221}{150} \times 200$$

$$= 369.33$$

Now,

$$S_k(B) = \frac{Q_3 + Q_1 - 2Md}{Q_3 - Q_1}$$

$$= \frac{648.076 + 194.375 - 2 \times 369.33}{648.076 - 194.375}$$

$$= 0.22$$

Since, $S_k(B) = 0.22 > 0$, the distribution is positively skewed.

2058 Q.No.6

Soln. Since, mid-value is given we should find their classes.

Calculation of Mean, Mode and Standard Deviation

Income	Mid-value(x)	No. of Workers(f)	$d' = \frac{X-450}{100}$	fd'	fd'2
100-200	-50 150 +50	80	-3	-240	720
200-300	$\frac{100}{2}$=50 250	105	-2	-210	420
300-400	350	120	-1	-120	120
400-500	A=450	165	0	0	0
500-600	550	100	1	100	100
600-700	650	90	2	180	360
700-800	750	60	3	180	540
800-900	850	40	4	160	640
		$N=\Sigma f = 760$		$\Sigma fd' = 50$	$\Sigma fd'^2 = 2900$

Complete Yourself as Question No. 2076 Q.N.16

2073 Old Q.N.8

Soln

Calculation of first four moments about Mean

Profit in lakh (Rs)	No. of Companies (f)	Mid-value X	fX	$\bar{x}=5$ $x=X-\bar{x}$	$f(x-\bar{x})$ (fx)	$f(x-\bar{x})^2$ (fx^2)	$f(x-\bar{x})^3$ (fx^3)	$f(x-\bar{x})^4$ (fx^4)
0-2	3	1	3	-4	-12	48	-192	768
2-4	5	3	15	-2	-10	20	-40	80
4-6	9	5	45	0	0	0	0	0
6-8	5	7	35	2	10	20	40	80
8-10	3	9	27	4	12	48	192	768
	$N=\Sigma f = 25$		$\Sigma fX = 125$		$\Sigma fx = 0$	$\Sigma fx^2 = 136$	$\Sigma fx^3 = 0$	$\Sigma fx^4 = 1696$

$$\text{Mean}(\bar{x}) = \frac{\Sigma fX}{N} = \frac{125}{25} = 5$$

calculation of first four moments about mean (central moments)

$$\mu_1 = \frac{\Sigma fx}{N} = \frac{0}{25} = 0$$

$$\mu_2 = \frac{\Sigma fx^2}{N} = \frac{136}{25} = 5.44$$

$$\mu_3 = \frac{\Sigma fx^3}{N} = \frac{0}{25} = 0$$

$$\mu_4 = \frac{\Sigma fx^4}{N} = \frac{1696}{25} = 67.84$$

For skewness

$$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = \frac{0}{(5.44)^3} = 0$$

Since, $\beta_1 = 0$, the given distribution is symmetrical.

For kurtosis

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{67.84}{(5.44)^2} = 2.29$$

Since, $\beta_2 = 2.29 < 3$, the given distribution is platykurtic.

2072 Old Q.N.8

Soln

Calculation of mean, variance and skewness

Class Interval	Frequency (f)	Mid-value X	fX	$\bar{x}=25$ $x = X-\bar{X}$	$f(X-\bar{X})$ (fx)	$f(X-\bar{X})^2$ (fx^2)	$f(X-\bar{X})^3$ (fx^3)	$f(X-\bar{X})^4$ (fx^4)
0-10	4	5	20	-20	-80	1600	-32000	640000
10-20	6	15	90	-10	-60	600	-6000	60000
20-30	10	25	250	0	0	0	0	0
30-40	6	35	210	10	60	600	6000	60,000
40-50	4	45	180	20	80	1600	32000	640,000
	$N = \Sigma f = 30$		$\Sigma fX = 750$		$\Sigma fx = 0$	$\Sigma fx^2 = 4400$	$\Sigma fx^3 = 0$	$\Sigma fx^4 = 1400000$

For Mean

$$\text{Mean } (\bar{X}) = \frac{\Sigma fX}{N} = \frac{750}{30} = 25$$

Calculation of first four moments about Mean (central moments)

$$\mu_1 = \frac{\Sigma fx}{N} = \frac{0}{30} = 0$$

$$\mu_2 = \frac{\Sigma fx^2}{N} = \frac{4400}{30} = 146.67$$

$$\mu_3 = \frac{\Sigma fx^3}{N} = \frac{0}{30} = 0$$

$$\mu_4 = \frac{\Sigma fx^4}{N} = \frac{1400{,}000}{30} = 46666.67$$

$$\text{Variance} = \mu_2 = 146.67$$

For skewness

$$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = \frac{0}{(146.67)^3} = 0$$

Since, $\beta_1 = 0$, the given distribution is symmetrical.

Analytical Answer Question:

2074 old Q.N.11

Soln

Calculation of skewness and kurtosis

Wages per hour (Rs.)	No. of workers (f)	Mid-value (X)	$d' = \frac{X-70}{20}$	fd'	fd'^2	fd'^3	fd'^4
0-20	5	10	-3	-15	45	-135	405
20-40	7	30	-2	-14	28	-56	112
40-60	16	50	-1	-16	16	-16	16
60-80	20	A=70	0	0	0	0	0
80-100	28	90	1	28	28	28	28
100-120	12	110	2	24	48	96	192
120-140	10	130	3	30	90	270	810
140-160	2	150	4	8	32	128	512
	$N = \Sigma f = 100$			$\Sigma fd' = 45$	$\Sigma fd'^2 = 287$	$\Sigma fd'^3 = 315$	$\Sigma fd'^4 = 2075$

Calculation of First Raw moments

$$\mu'_1 = \frac{\Sigma fd'}{N} \times h = \frac{45}{100} \times 20 = 9$$

$$\mu'_2 = \frac{\Sigma fd'^2}{N} \times h^2 = \frac{287}{100} \times (20)^2 = 1148$$

$$\mu'_3 = \frac{\Sigma fd'^3}{N} \times h^3 = \frac{315}{100} \times (20)^3 = 25,200$$

$$\mu'_4 = \frac{\Sigma f d'^4}{N} \times (h)^4 = \frac{2075}{100} \times (20)^4 = 3320000$$

Calculation of central moments

$$\mu_1 = \mu'_1 - \mu'_1 = 9 - 9 = 0$$

$$\mu_2 = \mu'_2 - (\mu'_1)^2$$
$$= 1148 - (9)^2$$
$$= 1067$$

$$\mu_3 = \mu'_3 - 3 \cdot \mu'_2 \cdot \mu'_1 + 2(\mu'_1)^3$$
$$= 25200 - 3 \times 1148 \times 9 + 2 \times (9)^3$$
$$= -4338$$

$$\mu_4 = \mu'_4 - 4 \cdot \mu'_3 \cdot \mu'_1 + 6 \cdot \mu'_2 \cdot (\mu'_1)^2 - 3 \cdot (\mu'_1)^4$$
$$= 33,20,000 - 4 \times 25200 \times 9 + 6 \times 1148 \times (9)^2 - 3 \times (9)^4$$
$$= 2951045$$

For skewness

$$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = \frac{(-4338)^2}{(1067)^3} = 0.0155$$

since, $\mu_3 < 0$, the given distribution is negatively skewed.

For kurtosis

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{2951045}{(1067)^2} = 2.5921$$

Since. $\beta_2 = 2.5921 < 3$, the given distribution is platykurtic.

2073 Q. No. 17

Soln

a. calculation of frequency distribution

Number of customers (N) = 50

Minimum waiting time (X_s) = 17

Maximum waiting time (X_L) = 72

Number of classes, K = 7

Using struge's rule, we have,

$$\text{class size}, h = \frac{\text{Range}}{\text{Number of classes}} = \frac{X_L - X_s}{K} = \frac{72-17}{7}$$

$$= 7.85 \approx 10$$

First class = 10-20

Last class = 70-80

Calculation of Frequency distribution

Waiting Time	Tally bars	Number of customers (f)
10-20	|||	3
20-30	~~||||~~ ~~||||~~ |	11
30-40	~~||||~~ ~~||||~~ ||	12
40-50	~~||||~~ ~~||||~~ |||	13
50-60	~~||||~~ |	6
60-70	||||	4
70-80	|	1
		$N = \Sigma f = 50$

b.

Calculation of skewness and kurtosis

Waiting Time	No. of customers (f)	Mid-value(x)	$d' = \frac{x-45}{10}$	fd'	fd'^2	fd'^3	fd'^4
10-20	3	15	-3	-9	27	-81	243
20-30	11	25	-2	-22	44	-88	176
30-40	12	35	-1	-12	12	-12	12
40-50	13	A=45	0	0	0	0	0
50-60	6	55	1	6	6	6	6
60-70	4	65	2	8	16	32	64
70-80	1	75	3	3	9	27	81
	$N = \Sigma f = 50$			$\Sigma fd' = -26$	$\Sigma fd'^2 = 114$	$\Sigma fd'^3 = -116$	$\Sigma fd'^4 = 582$

Calculation of Raw moments

$$\mu_1' = \frac{\Sigma fd'}{N} \times h = \frac{-26}{50} \times 10 = -5.2$$

$$\mu_2' = \frac{\Sigma fd'^2}{N} \times h^2 = \frac{114}{50} \times (10)^2 = 228$$

$$\mu_3' = \frac{\Sigma fd'^3}{N} \times h^3 = \frac{-116}{50} \times (10)^3 = -2320$$

$$\mu_4' = \frac{\Sigma fd'^4}{N} \times h^4 = \frac{582}{50} \times (10)^4 = 116400$$

Calculation of central moments

$$\mu_1 = \mu_1' - \mu_1'$$
$$= -5.2 - (-5.2) = 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$
$$= 228 - (-5.2)^2$$
$$= 200.96$$

$$\mu_3 = \mu_3' - 3.\mu_2'.\mu_1' + 2.(\mu_1')^3$$
$$= -2320 - 3 \times 228 \times (-5.2) + 2 \times (-5.2)^3$$
$$= 955.58$$

$$\mu_4 = \mu_4' - 4.\mu_3'.\mu_1' + 6.\mu_2'.(\mu_1')^2 - 3.(\mu_1')^4$$
$$= 116400 - 4 \times (-2320) \times (-5.2) + 6 \times 228 \times (-5.2)^2 - 3 \times (-5.2)^4$$
$$= 102941.24$$

For skewness:

$$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = \frac{(955.58)^2}{(200.96)^3} = 0.1125$$

$\because \mu_3 > 0$, the given distribution is positively skewed.

For kurtosis:

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{102941.24}{(200.96)^2} = 2.549$$

$\because \beta_2 = 2.549 < 3$, the given distribution is platykurtic

2071 Q.N.19

Sol^n

i. Calculation of skewness and kurtosis by using moment

Assets (in million)	No. of listed Companies (f)	Mid-value(X)	$d' = \frac{X-17.5}{5}$	fd'	fd'^2	fd'^3	fd'^4
0-5	20	2.5	-3	-60	180	-540	1620
5-10	25	7.5	-2	-50	100	-200	400
10-15	50	12.5	-1	-50	50	-50	50
15-20	40	A=17.5	0	0	0	0	0
20-25	20	22.5	1	20	20	20	20
25-30	15	27.5	2	30	60	120	240
	$N = \Sigma f = 170$			$\Sigma fd' = -110$	$\Sigma fd'^2 = 410$	$\Sigma fd'^3 = -650$	$\Sigma fd'^4 = 2330$

Calculation of Raw moments

$$\mu_1' = \frac{\Sigma fd'}{N} \times h = \frac{-110}{170} \times 5 = -3.24$$

$$\mu_2' = \frac{\Sigma fd'^2}{N} \times h^2 = \frac{410}{170} \times (5)^2 = 60.29$$

$$\mu_3' = \frac{\Sigma fd'^3}{N} \times h^3 = \frac{-650}{170} \times (5)^3 = -477.94$$

$$\mu_4' = \frac{\Sigma fd'^4}{N} \times h^4 = \frac{2330}{170} \times (5)^4 = 8566.18$$

Calculation of central moments

$$\mu_1 = \mu_1' - \mu_1'$$
$$= -3.24 - (-3.24) = 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$
$$= 60.29 - (-3.24)^2$$
$$= 49.79$$

$$\mu_3 = \mu_3' - 3.\mu_2'.\mu_1' + 2.(\mu_1')^3$$
$$= -477.94 - 3 \times 60.29 \times (-3.24) + 2 \times (-3.24)^3$$
$$= 40.05$$

$$\mu_4 = \mu_4' - 4.\mu_3'.\mu_1' + 6.\mu_2'.(\mu_1')^2 - 3.(\mu_1')^4$$
$$= 8566.18 - 4 \times (-477.94) \times (-3.24) + 6 \times 60.29 \times (-3.24)^2 - 3 \times (-3.24)^4$$
$$= 5838.88$$

For skewness

$$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = \frac{(40.05)^2}{(49.79)^3} = 0.0130$$

Since, $\beta_1 = 0.0130 > 0$, the given distribution is positively skewed.

For kurtosis

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{5838.88}{(49.79)^2} = 2.35$$

Since, $\beta_2 = 2.35 < 3$, the given distribution is platykurtic.

③ Calculation of coefficient of variation (c.v)

$$\text{Mean}(\bar{x}) = A + \frac{\Sigma fd'}{N} \times h$$

$$= A + \mu_1'$$

$$= 17.5 + (-3.24)$$

$$= 14.26$$

$$\text{Variance} = \mu_2$$

$$\therefore \text{Standard Deviation}(\sigma) = \sqrt{\mu_2}$$

$$= \sqrt{49.79}$$

$$= 7.0562$$

$$\text{Coefficient of Variation (c.v)} = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{7.0562}{14.26} \times 100$$

$$= 49.48\ \%$$

2075 Q.N.12

Sol^n

Given: The raw moments of value a are:

$A = 5$, $\mu'_1 = 2$, $\mu'_2 = 20$, $\mu'_3 = 40$, $\mu'_4 = 50$

Calculation of central moments

$\mu_1 = \mu'_1 - \mu'_1$

$= 2 - 2 = 0$

$\mu_2 = \mu'_2 - (\mu'_1)^2$

$= 20 - (2)^2$

$= 16$

$\mu_3 = \mu'_3 - 3.\mu'_2.\mu'_1 + 2.(\mu'_1)^3$

$= 40 - 3 \times 20 \times 2 + 2 \times (2)^3$

$= -64$

$\mu_4 = \mu'_4 - 4.\mu'_3.\mu'_1 + 6.\mu'_2.(\mu'_1)^2 - 3(\mu'_1)^4$

$= 50 - 4 \times 40 \times 2 + 6 \times 20 \times (2)^2 - 3 \times (2)^4$

$= 162$

For Mean:

Mean$(\bar{X}) = A + \mu'_1 = 5 + 2 = 7$

For standard Deviation:

Standard Deviation $(\sigma) = \sqrt{\mu_2} = \sqrt{16} = 4$

For skewness:

$$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = \frac{(-64)^2}{(16)^3} = 1$$

Since, $\mu_3 = -64 < 0$, the given distribution is negatively skewed.

For kurtosis:

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{162}{(16)^2} = 0.633$$

Since, $\beta_2 = 0.633 < 3$, the distribution is platykurtic.

2074 Q.N.16

Soln

Given:

$N = 100$, $\Sigma f dx = -14$, $\Sigma f d^2x = 154$, $\Sigma f d^3x = -62$
$\Sigma f d^4x = 490$

Calculation of first raw moments.

$$\mu_1' = \frac{\Sigma f dx}{N} = \frac{-14}{100} = -0.14$$

$$\mu_2' = \frac{\Sigma f d^2x}{N} = \frac{154}{100} = 1.54$$

$$\mu_3' = \frac{\Sigma f d^3x}{N} = \frac{-62}{100} = -0.62$$

$$\mu_4' = \frac{\Sigma f d^4x}{N} = \frac{490}{100} = 4.9$$

Calculation of first four central moments

$$\mu_1 = \mu_1' - \mu_1'$$
$$= -0.14 - (-0.14)$$
$$= 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$
$$= 1.54 - (-0.14)^2$$
$$= 1.5204$$

$$\mu_3 = \mu_3' - 3 \cdot \mu_2' \cdot \mu_1' + 2(\mu_1')^3$$
$$= -0.62 - 3 \times 1.54 \times (-0.14) + 2 \times (-0.14)^3$$
$$= 0.021312$$

$$\mu_4 = \mu_4' - 4 \cdot \mu_3' \cdot \mu_1' + 6 \cdot \mu_2' (\mu_1')^2 - 3(\mu_1')^4$$
$$= 4.9 - 4 \times (-0.62) \times (-0.14) + 6 \times 1.54 \times (-0.14)^2 - 3 \times (-0.14)^4$$
$$= 4.73275152$$

For skewness:

$$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = \frac{(0.021312)^2}{(1.5204)^3} = 0.0001$$

Hence, the given distribution is approximately symmetrical.

For kurtosis:

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{4.73275152}{(1.5204)^2} = 2.047$$

Since, $\beta_2 = 2.047 < 3$, the given distribution is platykurtic.

2075 Q.N.13

Soln

Arrange the data in ascending order

11, 15, 15, 17, 20, 21, 22, 23, 25, 29, 29

We have,

Smallest value $(X_S) = 11$

Largest value $(X_L) = 29$

$N = 11$

For Q_1

$$Q_1 = \text{Value of } \left(\frac{N+1}{4}\right)^{th} \text{ item}$$

$$= \text{Value of } \left(\frac{11+1}{4}\right)^{th} \text{ item}$$

$$= \text{Value of } 3^{rd} \text{ item}$$

$$= 15$$

For Median:

$$Md = \text{Value of } \left(\frac{N+1}{2}\right)^{th} \text{ item}$$

$$= \text{Value of } \left(\frac{11+1}{2}\right)^{th} \text{ item}$$

$$= \text{Value of } 6^{th} \text{ item}$$

$$= 21$$

For Q_3

$$Q_3 = \text{Value of } \left(\frac{3(N+1)}{4}\right)^{th} \text{ item}$$

$= \text{Value of } \left(\frac{3(11+1)}{4}\right)^{th} \text{ item}$

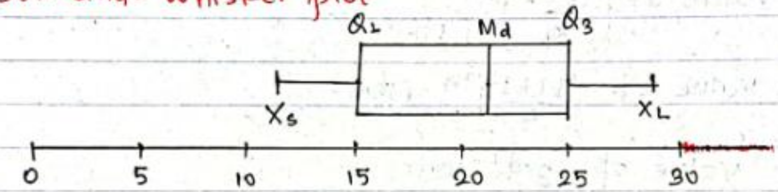
$= \text{Value of } 9^{th} \text{ item}$

$= 25$

Five Number summary is

X_s	Q_1	M_d	Q_3	X_L
11	15	21	25	29

Box-and-whisker plot



Since, the left side length is longer, so it is negatively skewed.

2072 (ii) Q.No.13

Soln

Arrange the given data in ascending order

264, 266, 298, 317, 342, 426, 451, 492, 512, 545, 562, 631, 1049

We have,

$N = 13$

Smallest value $(X_s) = 264$

Largest value $(X_L) = 1049$

For Q_1:

$$Q_1 = \text{Value of } \left(\frac{N+1}{4}\right)^{th} \text{ item}$$

$$= \text{Value of } \left(\frac{13+1}{4}\right)^{th} \text{ item}$$

$$= \text{Value of } 3.5^{th} \text{ item}$$

$$= 3^{rd} \text{ item} + 0.5\,(4^{th} \text{ item} - 3^{rd} \text{ item})$$

$$= 298 + 0.5(317 - 298)$$

$$= 307.5$$

For Median:

$$M_d = \text{Value of } \left(\frac{N+1}{2}\right)^{th} \text{ item}$$

$$= \text{Value of } \left(\frac{13+1}{2}\right)^{th} \text{ item}$$

$$= \text{Value of } 7^{th} \text{ item}$$

$$= 451$$

For Q_3:

$$Q_3 = \text{Value of } \left(\frac{3(N+1)}{4}\right)^{th} \text{ item}$$

$$= \text{Value of } \left(\frac{3(13+1)}{4}\right)^{th} \text{ item}$$

$$= \text{Value of } 10.5^{th} \text{ item}$$

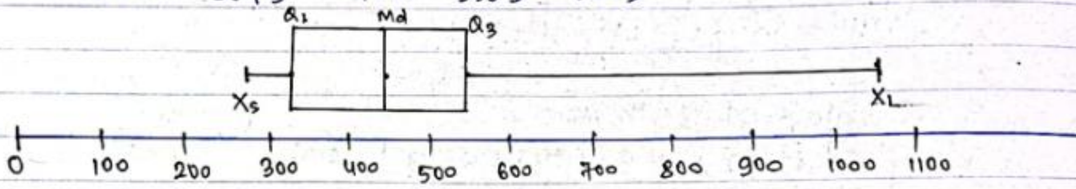
$$= 10^{th} \text{ item} + 0.5\,(11^{th} \text{ item} - 10^{th} \text{ item})$$

$$= 545 + 0.5(562 - 545)$$

$$= 553.5$$

Five number summary:

X_s	Q_1	M_d	Q_3	X_L
264	307.5	451	553.5	1049



c. Yes, the data are positevely skewed since it has longer tail on right side than that of left whisker.

2071 old Q.No.5

Sol^n

Given:

① Pearson's coefficient of skewness, $S_K(P) = 0.4$

Coefficient of variation (CV) = 30%

Mode (M_0) = 88

Mean ($\bar{X}$) = ?

Median (M_d) = ?

Here,

$$S_K(P) = \frac{\text{Mean} - \text{Mode}}{\text{S.D}}$$

$$0.4 = \frac{\bar{X} - 88}{0.3\bar{X}}$$

$$0.12\bar{X} = \bar{X} - 88$$

$$\therefore \bar{X} = 100$$

$$CV = \frac{\sigma}{\bar{X}} \times 100$$

$$\frac{30}{100} = \frac{\sigma}{\bar{X}}$$

$$\therefore \sigma = 0.3\bar{X} \quad \text{— ①}$$

Using empirical relationship, we have.

$$M_o = 3M_d - 2\bar{x}$$

$$88 = 3M_d - 2 \times 100$$

$$3M_d = 288$$

$$\therefore M_d = \frac{288}{3} = 96$$

Here, The value of mean and median are 100 and 96 respectively.

② Given:

$A = 4, \ \mu'_1 = 1, \ \mu'_2 = 4, \ \mu'_3 = 10$

$$\text{Mean } (\bar{x}) = A + \mu'_1 = 4 + 1 = 5$$

$$\text{Variance } (\sigma)^2 = \mu_2 = \mu'_2 - (\mu'_1)^2$$

$$= 4 - (1)^2$$

$$= 3$$

Third moment about mean

$$\mu_3 = \mu'_3 - 3 \cdot \mu'_2 \cdot \mu'_1 + 2 \cdot (\mu'_1)^3$$

$$= 10 - 3 \times 4 \times 1 + 2 \times (1)^3$$

$$= 10 - 12 + 2$$

$$= 0$$

Hence, the value of mean, variance and third moment about mean are 5, 3 and 0 respectively.

2070 Q.N.10 (i)

Soln

Given:

Standard deviation of a symmetrical distribution (σ) = 4

Fourth moment about mean, (μ_4) = ?

We know that:

Variance = $\mu_2 = (\sigma)^2 = (4)^2 = 16$

i. The distribution will be platykurtic if

$$\beta_2 < 3$$

$$\frac{\mu_4}{(\mu_2)^2} < 3$$

$$\frac{\mu_4}{(16)^2} < 3$$

$$\mu_4 < 768$$

∴ The value of fourth mount about mean should be less than 768 in order that the distribution is platykurtic.

ii. The distribution will be leptokurtic if

$$\beta_2 > 3$$

$$\frac{\mu_4}{(\mu_2)^2} > 3$$

$$\frac{\mu_4}{(16)^2} > 3$$

$$\therefore \mu_4 > 768$$

∴ The value of fourth moment about mean should be greater than 768, in order that the distribution is leptokurtic.

2065 Q.NO.3

Soln

Caln of Mean, Mode and Standard Deviation

Size in Inches	No. of observations (f)	Mid-Value (x)	$d' = \frac{X-37.5}{3}$	fd'	fd'^2
30-33	3	31.5	-2	-6	12
33-36	5	34.5	-1	-5	5
36-39	26 f_0	A = 37.5	0	0	0
$\boxed{39-42}$	46 → f_1	40.5	1	46	46
42-45	20 → f_2	43.5	2	40	80
45-48	10	46.5	3	30	90
	$N = \Sigma f = 110$			$\Sigma fd' = 105$	$\Sigma fd'^2 = 233$

For Mean:

$$\text{Mean}(\bar{X}) = A + \frac{\Sigma fd'}{N} \times h$$

$$= 37.5 + \frac{105}{110} \times 3$$

$$= 40.364$$

For Mode

Since, the highest frequency is 46, the modal class is 39-42. so

$L = 39$, $f_1 = 46$, $f_0 = 26$, $f_2 = 20$, $h = 3$

We have,

$$M_0 = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 39 + \frac{46-26}{2\times46-26-20} \times 3$$

$$= 40.304$$

For standard Deviation:

$$\text{Standard Deviation}(\sigma) = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times h$$

$$= \sqrt{\frac{233}{110} - \left(\frac{105}{110}\right)^2} \times 3$$

$$= 3.295$$

Pearson's Measure of skewness:

$$S_K(P) = \frac{\bar{X} - M_0}{\sigma}$$

$$= \frac{40.364 - 40.304}{3.295}$$

$$= 0.018$$

Since, $S_K(P) = 0.018 > 0$, the given distribution is positively skewed.

2070 Q.No. 10b

Soln

Arbitary value $(A) = 4$

First four raw moments: $\mu'_1 = 1$, $\mu'_2 = 3$, $\mu'_3 = 7$, $\mu'_4 = 21$

Central moments:

$\mu_1 = \mu'_1 - \mu'_1 = 0$

$\mu_2 = \mu'_2 - (\mu'_1)^2$

$= 3 - (1)^2 = 2$

Now,

Mean $(\bar{x}) = A + \mu'_1 = 4 + 1 = 5$

Standard Deviation $(\sigma) = \sqrt{\mu_2} = \sqrt{2} = 1.41$

2068 Q.No.7

Soln

Given:

Karl pearson's coefficient of skewness $= 0.5$

Median $= 42$

Mode $= 36$

Here,

$S_k(P) = \dfrac{\bar{X} - M_0}{\sigma}$

$0.5 = \dfrac{45 - 36}{\sigma}$

$\therefore \sigma = 18$

$M_0 = 3M_d - 2\bar{X}$

$36 = 3 \times 42 - 2\bar{X}$

$\therefore \bar{X} = 45$

Coefficient of Variation

$CV = \dfrac{\sigma}{\bar{X}} \times 100 = \dfrac{18}{45} \times 100$

$= 40\%$